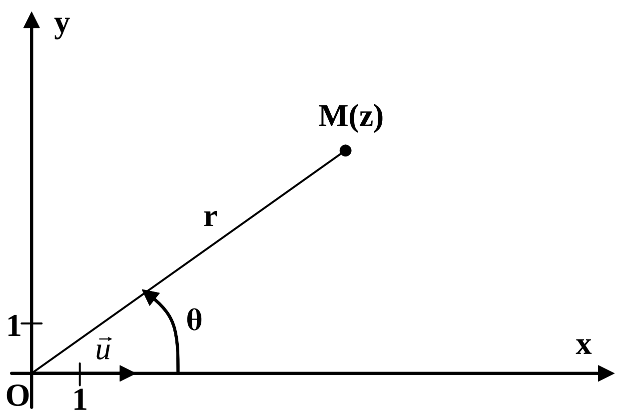
NOMBRES COMPLEXES

- Propriétés algébriques:
  - $i^2 = -1$
  - $z = a + ib$: **forme algébrique**, $\bar{z} = a - ib$: **conjugué**
  - x = Re(z), y = Im(z), $z + \bar{z} = 2\,\mathrm{Re}(z)$, $z - \bar{z} = 2i\,\mathrm{Im}(z)$
  - $\bar{\bar{z}} = z$, $z\bar{z} = a^2 + b^2$, $\dfrac{1}{a+ib} = \dfrac{a-ib}{a^2+b^2}$
  - z est **réel** si et seulement si $\bar{z} = z$
  - z est **imaginaire pur** si et seulement si $\bar{z} = -z$

- Forme trigonométrique:



- $|z| = OM$
- $\arg(z) = \left(\vec{u}, \overrightarrow{OM}\right) = \theta \ [2\pi]$
- $|z| = \sqrt{x^2 + y^2}$
- $|z' - z| = MM'$
- $z = z' \Leftrightarrow |z| = |z'|$ et $\theta = \theta' \ [2\pi]$
- z **réel** si et seulement si θ = 0 [π]
- z **imaginaire pur** si et seulement si $\theta = \dfrac{\pi}{2} \ [\pi]$

- Forme exponentielle:
  - $z = a + ib = re^{i\theta} = r(\cos\theta + i\sin\theta)$ avec $r = \sqrt{a^2 + b^2}$
  - $e^{i0} = 1$, $e^{i2\pi} = 1$, $e^{i\pi} = -1$, $e^{i\frac{\pi}{2}} = i$
  - $\cos\theta = \dfrac{e^{i\theta} + e^{-i\theta}}{2}$ et $\sin\theta = \dfrac{e^{i\theta} - e^{-i\theta}}{2i}$
  - $\left(e^{i\theta}\right)^n = e^{in\theta}$
  - $z = |z|(\cos\theta + i\sin\theta) = |z|e^{i\theta}$

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- Quelques cas particuliers importants:

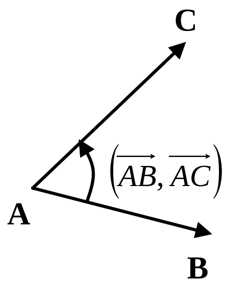
Nombre complexe	**Module**	**Un argument**	**Une forme exponentielle**
z	r	θ	$re^{i\theta}$
$\bar{z}$	r	$-\theta$	$re^{-i\theta}$
$-z$	r	$\theta+\pi$	$re^{i(\theta+\pi)}$
$\frac{1}{z}$	$\frac{1}{r}$	$-\theta$	$\frac{1}{r}e^{-i\theta}$
z^n	r^n	$n\theta$	$r^n e^{in\theta}$
zz'	rr'	$\theta+\theta'$	$rr'e^{i(\theta+\theta')}$
$\frac{z}{z'}$	$\frac{r}{r'}$	$\theta-\theta'$	$\frac{r}{r'}e^{i(\theta-\theta')}$
$k > 0 \in \Re$	k	0	ke^{i0}
$k < 0 \in \Re$	$\|k\|$	π	$\|k\|e^{i\pi}$

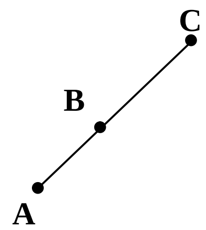
- Equation du second degré à coefficients réels: $az^2+bz+c=0$

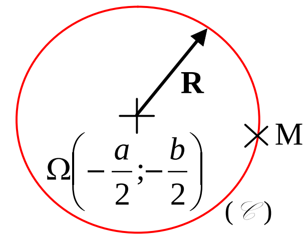
Si $\Delta = b^2 - 4ac < 0$, les solutions sont les complexes conjugués :

$$z = \frac{-b-i\sqrt{|\Delta|}}{2a} \text{ et } z' = \frac{-b+i\sqrt{|\Delta|}}{2a}$$

- Applications à la géométrie







$x^2 + ax + y^2 + by = 0$ est l'équation du cercle

$\mathscr{C}(\Omega,\ R=\sqrt{\frac{a^2}{4}+\frac{b^2}{4}})$

$$\left|\frac{c-a}{b-a}\right| = \frac{AC}{AB}$$

$$\arg\left(\frac{c-a}{b-a}\right) = (\overrightarrow{AB},\overrightarrow{AC})$$

$[2\pi]$

A, B, C **alignés**

$\Leftrightarrow (\overrightarrow{AB},\overrightarrow{AC}) = k\pi$

$\Leftrightarrow$

$$\arg\left(\frac{z_B - z_A}{z_C - z_A}\right) = k\pi$$

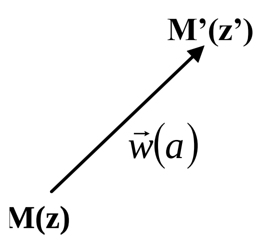
En effet : $x^2 + ax + y^2 + by = 0$

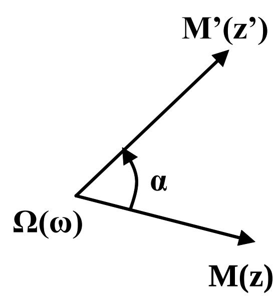
$$\Leftrightarrow \left(x+\frac{a}{2}\right)^2 + \left(y+\frac{b}{2}\right)^2 = \left(\frac{a^2}{4}+\frac{b^2}{4}\right) = R^2$$

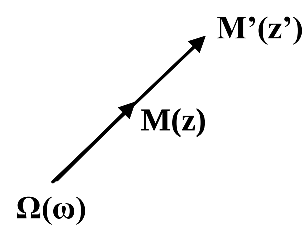
$$\Leftrightarrow |z_\Omega - z_M|^2 = R^2$$

$$\Leftrightarrow \Omega M^2 = R^2$$

- Transformations géométriques:







Translation de vecteur $\vec{w}(a)$:

$$z' = z + a$$

Rotation (Ω,α):

$$z' - \omega = e^{i\alpha}(z-\omega)$$

$$\Leftrightarrow \begin{cases} |z'-\omega| = |z-\omega| \\ \arg\left(\frac{z'-\omega}{z-\omega}\right) = \alpha[2\pi] \end{cases}$$

Homothétie (Ω, k):

$$\overrightarrow{\Omega M'} = k\overrightarrow{\Omega M}$$

$$\Leftrightarrow z' - \omega = k(z-\omega)$$

$$\Leftrightarrow \begin{cases} |z' - z_\Omega| = k|z - z_\Omega| \\ \arg\left(\frac{z'-z_\Omega}{z-z_\Omega}\right) = k\pi \end{cases}$$

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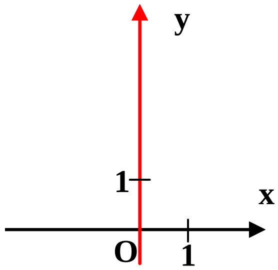
- Lieu de points:



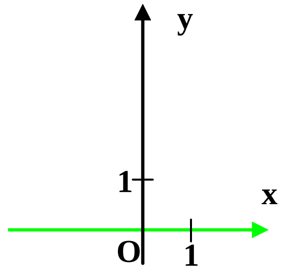
Z **imaginaire pur**

$\Leftrightarrow$ Re(Z) = 0

$\Leftrightarrow \arg Z = \frac{\pi}{2}[\pi]$

$\Leftrightarrow Z = |Z|e^{ik\frac{\pi}{2}}$

Z est représenté par l'axe des ordonnées



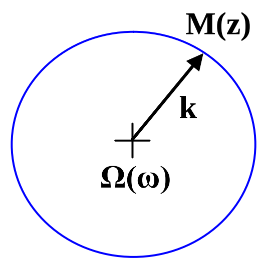
Z **réel**

$\Leftrightarrow$ Im(Z) = 0

$\Leftrightarrow \arg Z = 0[\pi]$

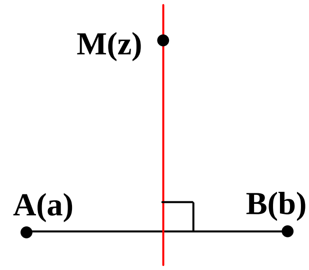
$\Leftrightarrow Z = |Z|e^{ik\pi}$

Z est représenté par l'axe des abscisses



M(z) ∈ 𝒞(Ω(ω), R = k)

$\Leftrightarrow |z-\omega| = k$



M(z) ∈ **médiatrice** de [AB]

$\Leftrightarrow |z-a| = |z-b|$

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