Solving of $\frac{5\left(x^4+3\,x^2-1\right)}{6} = 0$

$\frac{5\left(x^4+3\,x^2-1\right)}{6} = 0$ is equivalent to the fourth degree equation $ax^4 + bx^3 + cx^2 + dx + e = 0$ with:

$$\begin{array}{rcr} a & = & \frac{5}{6} \\ b & = & 0 \\ c & = & \frac{5}{2} \\ d & = & 0 \\ e & = & -\frac{5}{6} \end{array}$$

It is a bi-squared equation. We thus write $X = x^2$.

This takes us to the equation: $\frac{5\,X^2}{6} + \frac{5\,X}{2} - \frac{5}{6} = 0$. Its discriminant is equal to $\Delta = b^2 - 4ac = (\frac{5}{2})^2 - 4(\frac{5}{6})(-\frac{5}{6}) = \frac{325}{36} \simeq 9,03 > 0$.

This second degree equation has thus two solutions:

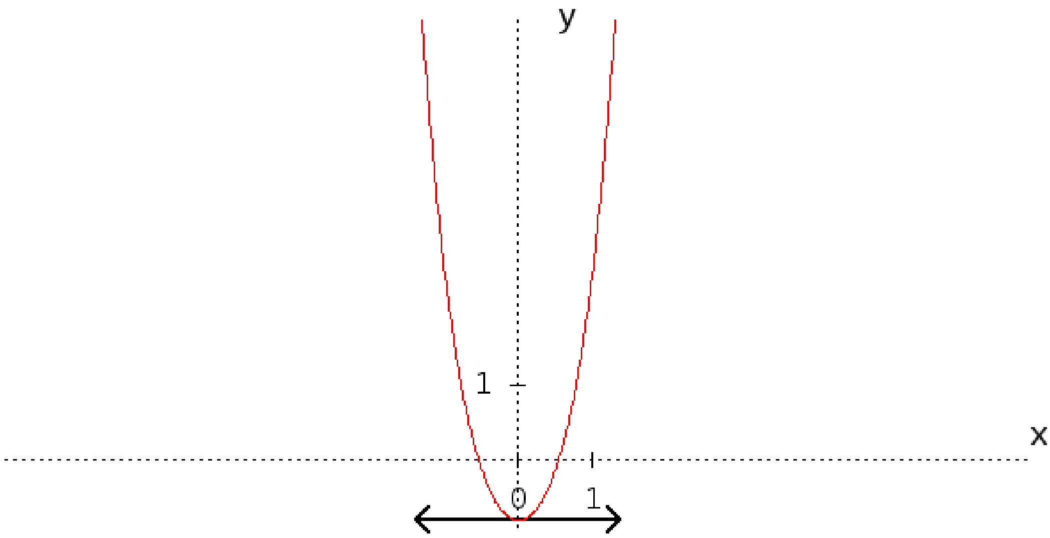
$$\begin{array}{rcccccc} X_1 & = & \frac{-b-\sqrt{\Delta}}{2a} & = & -\frac{\sqrt{13}}{2} - \frac{3}{2} & \simeq & -3,30 \\ X_2 & = & \frac{-b+\sqrt{\Delta}}{2a} & = & \frac{\sqrt{13}}{2} - \frac{3}{2} & \simeq & 0,303 \end{array}$$

Since $X_1 < 0$ and $X_2 > 0$, we get the real solutions:

$$\begin{array}{rcccccr} x_1 & = & \sqrt{X_2} & = & \frac{\sqrt{\sqrt{13}-3}}{\sqrt{2}} & \simeq & 0,550 \\ x_2 & = & -\sqrt{X_2} & = & -\frac{\sqrt{\sqrt{13}-3}}{\sqrt{2}} & \simeq & -0,550 \end{array}$$

All solutions in $\mathbb{R}$ are thus: $S = \{-\frac{\sqrt{\sqrt{13}-3}}{\sqrt{2}}; \frac{\sqrt{\sqrt{13}-3}}{\sqrt{2}}\}$.

Graphically, the curve having for equation $y = \frac{5\,x^4}{6} + \frac{5\,x^2}{2} - \frac{5}{6}$ is cutting the x axis two times, at points having for abscisse x_1 and x_2.



You can view dynamically the curve by clicking here: http://www.lovemaths.fr/curve_en.html (and then accept to run the java application).

Regarding complex solutions, we find in $\mathbb{C}$:

$$\begin{array}{rclcll} x_1 & = & \sqrt{X_2} & = & \frac{\sqrt{\sqrt{13}-3}}{\sqrt{2}} & \simeq & 0,550 \\ x_2 & = & -\sqrt{X_2} & = & -\frac{\sqrt{\sqrt{13}-3}}{\sqrt{2}} & \simeq & -0,550 \\ x_3 & = & i\sqrt{-X_1} & = & \frac{\sqrt{\sqrt{13}+3}\,i}{\sqrt{2}} & \simeq & 1,82i \\ x_4 & = & -i\sqrt{-X_1} & = & -\frac{\sqrt{\sqrt{13}+3}\,i}{\sqrt{2}} & \simeq & -1,82i \end{array}$$

Thus $S = \{\frac{\sqrt{\sqrt{13}+3}\,i}{\sqrt{2}}; -\frac{\sqrt{\sqrt{13}+3}\,i}{\sqrt{2}}; \frac{\sqrt{\sqrt{13}-3}}{\sqrt{2}}; -\frac{\sqrt{\sqrt{13}-3}}{\sqrt{2}}\}$.

Note: these results have been obtained from an automated program and are not guaranteed to be exact.